GENERALIZATION OF THE EQUATIONS FOR INTERSECTING TURBULENT JETS

Victor Pimsner Stefan Vintilă

1. Preliminary

The intersections of turbulent jets have a number of technical applications.

The intersecting jets may be isothermal and non-isothermal, as their temperatures are equal or differing from one another and from that of the medium where the intersection takes place.

In terms on one hand of the geometric, kinetic and heat parameters of the jets, and on the other hand of the initial incidence angle, the intersection may take place either between the jet boundary layers, or between their base spaces. The space where the intersection takes place may also affect both the characteristics of the incident jets and those of the jet resulting from intersection, and we may arrive, e.g. to an intersection of half-jets.

In the following we will refer to the isothermal and nonisothermal incompressible turbulent jets, plane-parallel, of gases in gas, with initial fields of uniform velocities or which are differing in value from one another, intersecting under any angle of incidence in an unbounded space.

Previous works /3/, /4/ have suggested a calculation form and have determined the general equation of the plane-parallel, isothermal and non-isothermal turbulent jet intersection in the above mentioned conditions; the main magnitudes describing the intersection, i.e.: equation of the resulting jet round axis; variations of the axial velocity and of the resulting jet half-width, as well as temperature axial difference have been determined.

Starting from the plane-parallel, non-isothermal turbulent jet intersection results we have obtained in this paper through particularization the results for the plane-parallel, isothermal turbulent jet intersection and further on, through boundary transition, when the slit width of one of the jets tends to infinite, we have reached the result known from literature /1/, /5/ of the intersection of a turbulent jet with an uniform current.

The high generalization degree of the plane-parallel, nonisothermal turbulent jet intersection equations is thus emphasized. Besides, it should be mentioned that a series of experimental results have been obtained which are confirming the calculation form and the adopted fundamental assumption.

2. General equations of the plan-parallel, non-isothermal turbulent jet intersection

For studying the intersection we have used the system of reference rectangular axes xoy (Fig.1) and for each jet the systems: x_1oy_1, obteined from xoy through a rotation of the angle α_0, and $x_2o_2y_2$, respectively, obtained from xoy through a translatory motion in the positive sense of the axis of ordinates, by the value λ, where ox_1 and o_2x_2, respectively, coincide with the respective jet initial axes.

The angle θ of the jet lateral diffusion (Fig.1) being known /1/, /2/, we may write the equations of the lateral limits O_1A and O_2A, respectively, of the

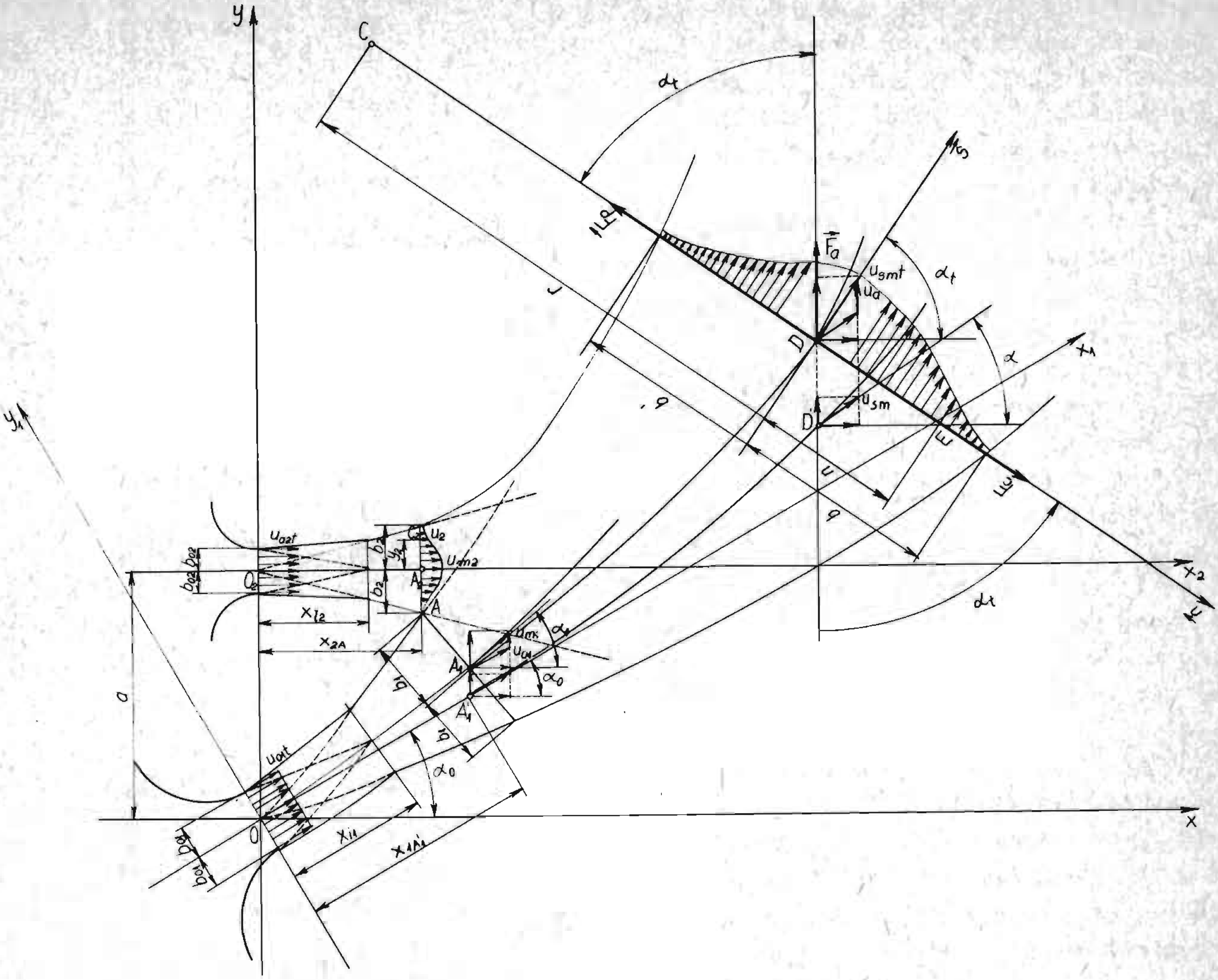

FIGURE 1. INTERSECTION OF NON-ISOTHERMAL PLANE-PARALLEL

two jets and from their intersection we obtain the coordinates of point A (Fig.1):

$$x_A = \lambda \frac{1}{tg\theta + tg\,(\alpha_0 + \theta)}$$

$$y_A = \lambda \frac{tg(\alpha_0 + \theta)}{tg\theta + tg\,(\alpha_0 + \theta)}$$

By passing through the point A a plane normal on each of the axes ox_1, o_2x_2, respectively, of the two incident jets and a plane normal to the resultant jet curved axis, a control surface is formed with its lateral surfaces, enclosing a space for which the general equations of the intersection are written.

For this purpose the following assumptions are stated:

- the form of the cross section normal on the round axis of the jet resulting from intersection is considered flat, and rectangular;

- the jet half-widths after intersection, b and b', respectively, (Fig.1) will differ as a result of the asymmetric motion; but, in a first approximation, these half-widths are considered identical;

- the components of the velocity vectors on the normal to the round axis of the intersection resulting jet are smaller as compared to the components on the tangent $\vec{s}$ to the jet round axis in the same point.

The geometrical and physical magnitudes will be denoted by the subscripts 1 and 2, respectively, for the two jets engaged in intersection and the subscript s for the resulting jet; moreover, the subscript o will be used for the magnitudes related to the initial sections of the two jets and the subscript t for the thermal (non-isothermal) jets.

We consider that the intersection occurs between the base spaces of the two jets and that it has uniform but unequal initial velocity fields (e.g. $u_{01} > u_{02}$), so that the inclined jet (Fig.1) will have a total energy greater than the horizontal jet whom it will bend by drawing along. Starting from the mix initial sections normal on the ox_1 and o_2x_2 axes, respectively, and passing through the point A (Fig.1), a turbulent diffusion of substance, pulses and heat will be generated, the inclined jet acting on the horizontal jet with a pressure force located in the plane normal on the curved axis of the resulting jet. Owing to temperature differences, a non-uniformity of the density field will be generated and lifting forces (buoyancy) will arise both in each non-isothermal jet and in the jet resulting from intersection. The lifting forces from the incident jets may be neglected in comparison with the lifting force from the resulting jet, if the intersection occurs at the beginning of the incident jet base spaces. Preservation of the resulting jet on the round path will result from the action of the centrifugal force located in the plane normal to this path. The eddy dynamic equilibrium in the resulting jet will be established when the resultant of the pressure dF_p, lifting dF_a and centrifuge dF_c forces will vanish (Fig.1).

These being the conditions, the intersection of the non-isothermal plan-parallel turbulent jets will be described by the following system of equations:

- continuity equation:

$$\rho_s u_{st} dn = \rho_1 u_{1t} dy_1 + \rho_2 u_{2t} dy_2 \qquad (1)$$

- impulse equations in projection on the ox and oy axes, respectively:

$$\rho_s u_{st}^2 \cos\alpha_t \, dn = \rho_1 u_{1t}^2 \cos\alpha_o dy_1 + \rho_2 u_{2t}^2 dy_2 \qquad (2)$$

$$\rho_s u_{st}^2 \sin\alpha_t dn = \rho_1 u_{1t}^2 \sin\alpha_o \, dy_1 \qquad (3)$$

- energy equation:

$$c_s \rho_s u_{st} (T_s - T_i) dn = c_1 \rho_1 u_{1t} (T_1 - T_i) dy_1 + c_2 \rho_2 u_{2t} (T_2 - T_i) dy_2 \qquad (4)$$

- equation of dynamic equilibrium:

$$dF_p + dF_a \cos\alpha_t = - dF_c \qquad (5)$$

where:

$$dF_p = C_n \frac{(\rho_2 u_{2t}^2)_{mean}}{2} h \sin^2\alpha_t ds \qquad (6)$$

$$dF_a = (\rho_i - \rho_{s_{mean}}) g dV = (\rho_i - \rho_{s_{mean}}) \cdot g 2bhds \qquad (7)$$

$$dF_c = \frac{(\rho_s u_{st}^2)_{mean}}{r} 2bhds \qquad (8)$$

- equation of state:

$$p = \rho g RT \qquad (9)$$

The following symbols have been used in the above equations: u_{1t}; u_{2t}; u_{st} - velocities; ρ_1; ρ_2; ρ_s - densities; T_1; T_2; T_s - temperatures; c_1; c_2; c_s - specific heat capacities, in a point of the normal cross sections and on the incident jet axes and on the resulting jet round axis, respectively; T_i - absolute temperature of the medium where the intersection is generated;

α_t - local gradient of the round axis of the jet resulting from intersection, also referred to as angle of deflection;

C_n - factor allowing for the variation of the pressure exerted by the jet inclined on the contact surface with the horizontal jet;

dV - considered elementary space

R - gas specific constant;

g - gravity acceleration.

From the equation system (1) to (9) we have attained the differential equation:

$$y'^3 + Ay' + By'' = 0 \qquad (1o)$$

which integrated at boundary conditions (Fig.1):

- for $x=x_{A_1}$ and $y=y_{A_1}$: $y'_{A_1}=z_{A_1}=tg\alpha_o$

leads to the equation of the non-isothermal jet round axis resulting from the intersection of the two plane-parallel non-isothermal turbulent jets:

$$y=y_{A_1}+\frac{B}{\sqrt{A}}\left[arc\ tg\left(\pm\sqrt{(1+Actg^2\alpha_o)e^{\frac{2A}{B}(x-x_{A_1})}-1}\right)-\right.$$

$$\left.-arctg(\sqrt{A}\ ctg\alpha_o)\right] \qquad (11)$$

The magnitudes A and B from equations (1o) and (11) may be calculated by means of the known relations from jet conventional theory /1/ and of the following expressions /4/:

$$A=\frac{4g}{C_n}\cdot\frac{K_s}{K_2}\cdot\frac{1}{T_i}\cdot\frac{k_1b_1\rho_{1_{mean}}u_{o1}\left(\frac{u_{mt1}}{u_{o1}}\right)+k_2b_2\rho_{2_{mean}}u_{o2}\left(\frac{u_{mt2}}{u_{o2}}\right)}{k_2\rho_{2_{mean}}u_{o2}^2\left(\frac{u_{mt2}}{u_{o2}}\right)^2 b_2}\ \times$$

$$\times\ \frac{k_{1t}k_{1_{\Delta T}}K_{1_{\Delta T}}b_1\frac{c_{1_{mean}}}{c_{s_{mean}}}\rho_{1_{mean}}u_{o1}\Delta T_{o1}\left(\frac{u_{mt1}}{u_{o1}}\right)\left(\frac{\Delta T_{m1}}{\Delta T_{o1}}\right)}{k_{1t}K_1b_1\rho_{1_{mean}}u_{o1}^2\left(\frac{u_{mt1}}{u_{o1}}\right)^2\cos\alpha_o+k_{2t}K_2b_2\rho_{2_{mean}}u_{o2}^2\left(\frac{u_{mt2}}{u_{o2}}\right)^2}\ +$$

$$+\ \frac{k_{2t}k_{2_{\Delta T}}K_{2_{\Delta T}}b_2\frac{c_{2_{mean}}}{c_{s_{mean}}}\rho_{2_{mean}}u_{o2}\Delta T_{o2}\left(\frac{u_{mt2}}{u_{o2}}\right)\left(\frac{\Delta T_{m2}}{\Delta T_{o2}}\right)}{k_{1t}K_1b_1\rho_{1_{mean}}u_{o1}^2\left(\frac{u_{mt1}}{u_{o1}}\right)^2\cos\alpha_o+k_{2t}K_2b_2\rho_{2_{mean}}u_{o2}^2\left(\frac{u_{mt2}}{u_{o2}}\right)^2} \qquad (12)$$

$$B=\frac{4}{C_n}\cdot\frac{K_1}{K_2}\cdot\frac{\rho_{1_{mean}}}{\rho_{2_{mean}}}\cdot\left(\frac{u_{o1}}{u_{o2}}\right)^2\frac{\left(\frac{u_{1t_{mean}}}{u_{o1}}\right)^2}{\left(\frac{u_{2t_{mean}}}{u_{o2}}\right)^2}\ b_1\sin\alpha_o \qquad (13)$$

3. General equations of the plane-parallel, isothermal turbulent jet intersection

The system of axes of reference and the relative positions of the incident isothermal plane-parallel, turbulent jets engaged in intersection (Fig.2) are preserved as with the non-isothermal jets. The calculation form is also preserved, escept that the jets being isothermal the buoyancy type lifting forces are vanishing, hence the dynamic equilibrium of the turbulent flow in the rezulting jet will be established when the resultant of the pressure and centrifugal forces will vanish.

Therefore, the intersection of the plane-parallel, isothermal turbulent jets will be described by the equation system consisting of:

- continuity equation:

$$\rho_s u_s dn = \rho_1 u_1 dy_1 + \rho_2 u_2 dy_2 \qquad (14)$$

- impulse equations in projection on the ox and oy axes, respectively:

$$\rho_s u_s^2 \, dn \cos\alpha = \rho_1 u_1^2 \, dy_1 \cos\alpha_0 + \rho_2 \, u_2^2 dy_2 \qquad (15)$$

$$\rho_s \, u_s^2 \, dn \sin\alpha = \rho_1 \, u_1^2 dy_1 \sin\alpha_0 \qquad (16)$$

- equation of flow dynamic equilibrium:

$$dF_p = - dF_c \qquad (17)$$

or:

$$C_n \frac{\rho_2 (u_2^2)_{mean}}{2} h \sin^2\alpha \, ds = - \frac{\rho_s (u_s^2)_{mean} \, 2bhds}{r} \qquad (18)$$

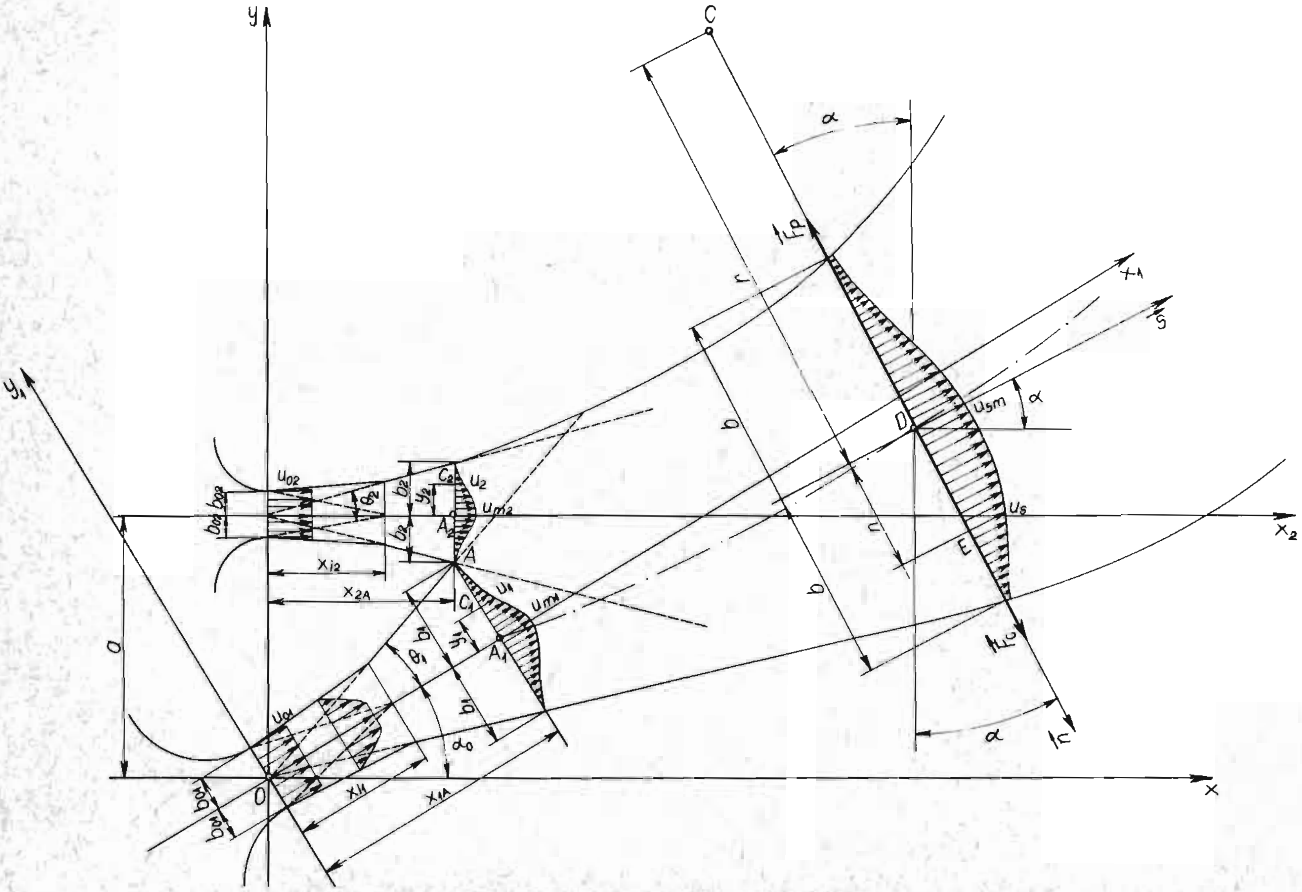

FIGURE 2. INTERSECTION OF TWO PLANE-PARALLEL ISOTHERMAL JETS

where the symbols have the known significances.

From the equation system (14) to (18) we have obtained the differential equation:

$$\frac{y'^3}{y''} = - F \qquad (19)$$

which integrated at boundary condition:

– for $x=x_{A_1}$ and $y=y_{A_1}$, $y'_{A_1} = tg\alpha_o$

gives the equation of the axis of the jet resulting from the plane-parallel, isothermal turbulent jet intersection:

$$y=y_{A_1}+F(\pm\sqrt{\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_o}-ctg\alpha_o) \qquad (2o)$$

where the magnitude F may be calculated /3/ by using the known relations from the jet conventional theory /1/ and has the following expression:

$$F = \frac{4}{C_n}\cdot\frac{K_1}{K_2}\cdot\frac{\rho_1}{\rho_2}\cdot\left(\frac{u_{o1}}{u_{o2}}\right)^2\cdot\frac{\left(\frac{u_{mean_1}}{u_{o1}}\right)^2}{\left(\frac{u_{mean_2}}{u_{o2}}\right)^2} b_1 \sin\alpha_o \qquad (21)$$

4. Intersection of the plane-parallel, isothermal turbulent jets, special case of the plane-parallel, non-isothermal turbulent jet intersection

The equation (2o) of the round axis of the jet resulting from the intersection of two plane-parallel, isothermal turbulent jets may be obtained from the equation (11) of the jet resulting from the intersection of non-sothermal jets, by passing this equation to boundary, when the two jets have identical temperatures. In this case, we notice from the relation (12) that $A \equiv 0$, and by comparing the relations (13) and (21) there directly results that $B \equiv F$.

Therefore, passing the equation (11) to boundary conditions we obtain:

$$y=y_{A_1}+\lim_{A\to 0}\frac{F}{\sqrt{A}}\cdot$$

$$arctg(\pm\sqrt{(1+A\,ctg^2\alpha_o)\,e^{\frac{2A}{F}(x-x_{A_1})}-1}) -$$

$$- arctg(\sqrt{A}\,ctg\,\alpha_o) \qquad (22)$$

It should be noticed that the boundary of the second term from the right-hand side of the relation (22) leads to the indeterminacy $\frac{0}{0}$, for whose elimination the rule of L'Hôspital should be applied and we obtain:

$$\frac{ctg^2\alpha_o\,e^{\frac{2A}{F}(x-x_{A_1})}+(1+A\,ctg^2\alpha_o)\cdot\frac{2}{F}(x-x_{A_1})\,e^{\frac{2A}{F}(x-x_{A_1})}}{2\sqrt{(1+A\,ctg^2\alpha_o)\,e^{\frac{2A}{F}(x-x_{A_1})}-1}}$$

$$\lim_{A\to 0}\left[\frac{\qquad}{2\sqrt{(1+A\,ctg^2\ _o)\,e^{\frac{2A}{F}(x-x_{A_1})}-1}}-\frac{\frac{ctg\alpha_o}{2\sqrt{A}}}{1+A\,ctg^2\alpha_o}\right]$$

$$y=y_{A_1}+F\cdot\frac{\qquad}{\lim_{A\to 0}\frac{1}{2\sqrt{A}}} \qquad (23)$$

or:

$$x=y_{A_1}+F\lim_{A\to 0}\frac{\frac{2}{F}(x-x_{A_1})(1+A\,ctg^2\alpha_o)+ctg^2\alpha_o}{1+A\,ctg^2\alpha_o}\cdot$$

$$\sqrt{\frac{A}{(1+A\,ctg^2\alpha_o)\,e^{\frac{2A}{F}(x-x_{A_1})}-1}} -$$

$$- \lim_{A\to 0}\frac{ctg\alpha_o}{1+A\,ctg^2\alpha_o} \qquad (24)$$

wherefrom there results:

$$y=y_{A_1}+F\left\{\left[\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_0\right]\cdot \lim_{A\to 0}\sqrt{\frac{A}{(1+A\,ctg^2\alpha_0)e^{\frac{2A}{F}(x-x_{A_1})}-1}}-ctg\alpha_0\right\} \quad (25)$$

It should be noticed that the radical boundary again leads to the indeterminacy $\frac{0}{0}$, requiring once more the application of the rule of L'Hôspital; we get:

$$y=y_{A_1}+F\left\{\left[\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_0\right]\cdot\sqrt{\frac{1}{\lim_{A\to 0}\ ctg^2\alpha_0 e^{\frac{2A}{F}(x-x_{A_1})}+\frac{2}{F}(x-x_{A_1})(1+A\,ctg^2\alpha_0)e^{\frac{2A}{F}(x-x_{A_1})}}}-ctg\alpha_0\right\} \quad (26)$$

wherefrom:

$$y=y_{A_1}+F\left[\frac{\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_0}{\sqrt{\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_0}}-ctg\alpha_0\right] \quad (27)$$

or

$$y=y_{A_1}+F\left[\pm\sqrt{\frac{2}{F}(x-x_{A_1})+ctg^2\alpha_0}-ctg\alpha_0\right] \quad (28)$$

which is just the equation (2o) of the round axis of the jet resulting from the intersection of the plane-parallel isothermal turbulent jets.

5. Intersection of a turbulent jet with an uniform flow, special case of jet intersection

Satrting from the equation (28) of the round axis of the jet resulting from the intersection of two plane-parallel, isothermal turbulent jets, at boundary, when the width b_{o2} of the horizontal jet slit tends to infinity ($b_{o2}\to\infty$), the even u_{o2} velocity distribution being preserved, we obtain the equation of the turbulent jet intersected by an even flow, equation that has been determined by Abramovici G.N./1/ according to the Volinski's method.

Actually, at boundary the equation (28) may be written as follows:

$$\lim_{b_{o2}\to\infty}\frac{y-y_{A_1}}{F}=\lim_{b_{o2}\to\infty}\left[\pm\sqrt{2\frac{x-x_{A_1}}{F}+ctg^2\alpha_0}-ctg\alpha_0\right] \quad (29)$$

But, from Fig.2 we see that for $b_{o2}\to\infty$ $x_{A_1}\to 0$ and $y_{A_1}\to 0$, such that relation (29) becomes:

$$\frac{y}{\lim\limits_{b_{o2}\to\infty} F}=\pm\sqrt{2\frac{x}{\lim\limits_{b_{o2}\to\infty} F}+ctg^2\alpha_0}-ctg\alpha_0 \quad (3o)$$

Therefore, only $\lim\limits_{b_{o2}\to\infty} F$ has still to be calculated, and with this end in view we will first correlate some symbols, i.e. : when $b_{o2}\to\infty$, $u_{o2}\to w$ where: w -is the uniform flow velocity and $u_{o1}\to u_o$ and $\rho_1\to\rho_o$ and $\rho_2\to\rho_w$ where: w; u_o; ρ_o; ρ_w.

are the symbols used by Abramovici G.N. /1/.

Since the distributions of the velocities u_o and w are uniform at boundary, we obtain:

$$\lim_{b_{o2}\to\infty} u_{mean_1} = u_o ; \quad \lim_{b_{o2}\to\infty} u_{mean_2} = w$$

By means of these observations, we obtain:

$$\lim_{b_{o2}\to\infty} F = \lim_{b_{o2}\to\infty} \frac{4}{C_n} \cdot \frac{K_1}{K_2} \cdot \frac{\varsigma_1}{\varsigma_2} \cdot \left(\frac{u_{o1}}{u_{o2}}\right)^2 \cdot$$

$$\frac{\left(\frac{u_{mean_1}}{u_{o1}}\right)^2}{\left(\frac{u_{mean_2}}{u_{o2}}\right)^2} - b_{o1} \sin\alpha_o = \frac{2\delta_o \varsigma_o u_o^2 \sin\alpha_o}{C_n \varsigma_w w^2} = \frac{2}{k} \quad (31)$$

where k is a constant determined by Abramovici:

$$k = \frac{C_n \varsigma_w w^2}{\delta_o \varsigma_o u_o^2 \sin\alpha_o} \quad (32)$$

where $\delta_o = b_{o1}$ and substituting the relation (31) in the equation (3o), we obtain:

$$y = \frac{2}{k}\left(\pm\sqrt{kx + \mathrm{ctg}^2\alpha_o} - \mathrm{ctg}\alpha_o\right) \quad (33)$$

which is just the equation of the round axis of a turbulent jet intersected by an uniform flow, equation that has been obtained by Abramovici G.N. /1/.

6. Conclusions

The equations of the intersections of isothermal and nonisothermal turbulent jets generalize a number of cases of intersections of jets with uniform flows.

Allowing for the stated assumptions, the obtained equations are valid for plane-parallel subsonic, isothermal and non-isothermal turbulent jets, respectively, of incompressible fluids.

The carried out experimental determinations have confirmed the accepted calculation form and assumptions.

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